**Module 3**

1. Find the maximum and minimum value of the function

x3 - 3x2 - 9x + 12

Solution:

f(x) = x3 - 3x2 - 9x + 12 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)

On diff. both sides we get,

f’(x) = 3x2 – 6x – 9

for Maxima and minima,

put f’(x) = 0,

3x2 – 6x – 9 = 0

x2 – 2x – 3 = 0

x2 – 3x + x – 3 = 0

x (x-3) + 1 (x-3) = 0

(x-3) (x+1) = 0

X = 3 or x = -1

Now diff eqn (1) wrt x we get,

f” (x) = 6x – 9

case 1:

when x = 3,

f” (3) = 6 \* 3 – 9 = 9

f” (3) > 0

f(x) is minimum at x = 3

Minimum value = f (3)

(3)3 – 3(3)2 - 9 \* 3 +12

27 – 27 – 27 + 12

= -15

Minimum value of f(x) is -15

Case 2:

When x = -1,

f” ( -1) = 6 \* (-1) – 9 = - 6 – 9 = -15

f” ( -1) < 0

f(x) is maximum at x = -1

Maximum value = f (-1)

(-1)3 – 3(-1)2 - 9 \* (-1) +12

(-1) – 3 + 9 + 12

= 17

Maximum value of f(x) is 17

1. Calculate the slope and the equation of a line which passes through the points (-1, -1) (3,8)

Solution:

Slop m of a line passing through two points (x1, y1) and (x2, y2) is given by,

m = (y2 – y1) / (x2 – x1)

then the slop of the line passing through the points (-1, -1) and (3, 8) is,

m = 8 - (- 1) / 3 – ( -1)

m = 9 / 4

m = 2.25

1. Solve for w’(z) when



Solution:

Quotient Rule:

dy / dx = [v \* du/dx – u \* dv/dx] / v2

w(z) = (4z – 5) / (2 – z)

w’ (z) = [(4z – 5) ( -1) – (2-z) \* 4] / (2-z)2

w’ (z) = [(5 – 4z) – (8 – 4z)] / (2-z)2

w’ (z) = [5 – 4z – 8 + 4z] / (2-z)2

w’ (z) = -3 / (2-z)2

1. Consider Y(x)= . Identify the critical values and verify if it gives maxima or minima.

Solution:

Y(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)

Y’(x) = 6x2 + 12x + 3 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)

Y” (x) = 12x + 12 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)

Critical points,

Putting Eqn (2) = 0,

Y’(x) = 6x2 + 12x + 3 = 0

Y’(x) = 2x2 + 4x + 1 = 0

x = -1(-1/ (1/√2)) or x = -1(+1/ (1/√2))

Maxima or Minima,

At x = -1(-1/ (1/√2)),

From eqn (3),

Y” (-1(-1/ (1/√2))) = 12 ( -1 / √2) < 0

Y(x) has maximum value at x = -1(-1/ (1/√2))

At x = -1(+1/ (1/√2)),

From eqn (3),

Y” (-1(+1/ (1/√2))) = 12 (+1 / √2) > 0

Y(x) has minimum value at x = -1(+1/ (1/√2))

1. Determine the critical points and obtain relative minima or maxima or saddle points of function f defined by



Y = 2x12 + 2x1x2 + 2x22 + 6x1

∂y / ∂x1 = 4x1 + 2x2 + 6

∂y / ∂x2 = 2x1 + 4x2

Critical points,

∂y / ∂x1 = 0 and ∂y / ∂x2 = 0,

∂y / ∂x2 = 2x1 + 4x2 = 0

x1 + 2x2 = 0

x2 = - (1/2) x1

Substituting x2 in ∂y / ∂x1

∂y / ∂x1 = 4x1 + 2x2 + 6 = 0

= 4x1 + 2 (- (1/2) x1) + 6

= 4x1 – x1 +6

x1 = -2

Substituting x1 in x2

x2 = - (1/2) x1

x2 = - (1/2) (-2)

x2 = 1

Critical points are, x1 = -2 and x2 = 1

Saddle Points,

f1 = ∂y / ∂x1 = 4x1 + 2x2 + 6

f2 = ∂y / ∂x2 = 2x1 + 4x2

Second order direct partials,

∂2y/∂x12 = f11 = 4

∂2y/∂x22 = f22 = 4

Second order Cross partials,

∂2y/∂x1∂x2 = f12 = 2

∂2y/∂x2∂x1 = f21 = 2

By using hessian determinant,

|H| = 16-4 = 12 is Saddle point.